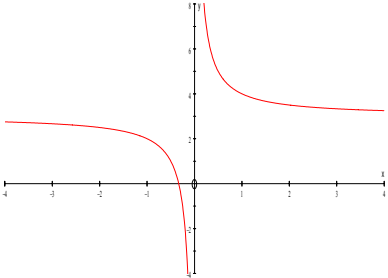
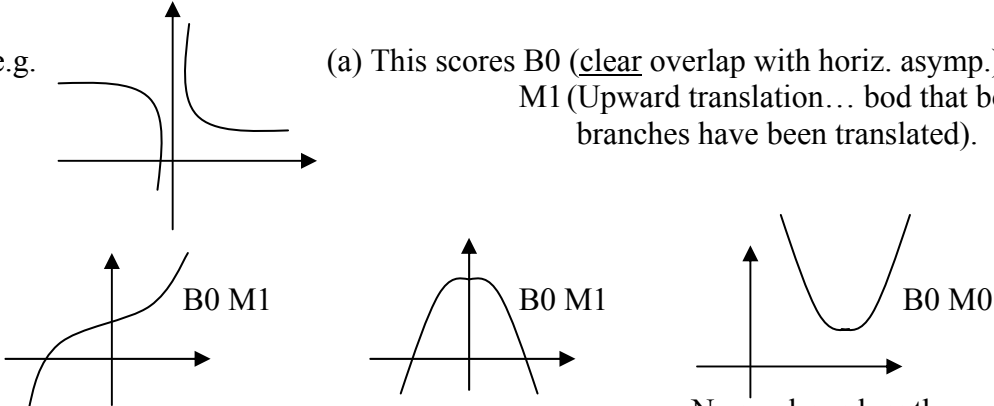


January 2007  
6663 Core Mathematics C1  
Mark Scheme

Question Number	Scheme	Mark
1.	$4x^3 \rightarrow kx^2$ or $2x^{\frac{1}{2}} \rightarrow kx^{-\frac{1}{2}}$ ( $k$ a non-zero constant) $12x^2, + x^{-\frac{1}{2}} \dots\dots, (-1 \rightarrow 0)$	M1 A1, A1, B1 (4) <b>4</b>
	<p>Accept equivalent alternatives to <math>x^{-\frac{1}{2}}</math>, e.g. <math>\frac{1}{x^{\frac{1}{2}}}</math>, <math>\frac{1}{\sqrt{x}}</math>, <math>x^{-0.5}</math>.</p> <p>M1: <math>4x^3</math> ‘differentiated’ to give <math>kx^2</math>, or...  <math>2x^{\frac{1}{2}}</math> ‘differentiated’ to give <math>kx^{-\frac{1}{2}}</math> (but not for just <math>-1 \rightarrow 0</math>).</p> <p>1<sup>st</sup> A1: <math>12x^2</math> (Do not allow just <math>3 \times 4x^2</math>)</p> <p>2<sup>nd</sup> A1: <math>x^{-\frac{1}{2}}</math> or equivalent. (Do not allow just <math>\frac{1}{2} \times 2x^{-\frac{1}{2}}</math>, but allow <math>1x^{-\frac{1}{2}}</math> or <math>\frac{2}{2}x^{-\frac{1}{2}}</math>).</p> <p>B1: <math>-1</math> differentiated to give zero (or ‘disappearing’). Can be given provided that at least one of the other terms has been changed.                      Adding an extra term, e.g. <math>+ C</math>, is B0.</p>	

Question Number	Scheme	Marks
2.	<p>(a) <math>6\sqrt{3}</math> <span style="margin-left: 150px;"><math>(a = 6)</math></span></p> <p>(b) Expanding <math>(2 - \sqrt{3})^2</math> to get 3 or 4 separate terms</p> <p style="margin-left: 40px;"><math>7, -4\sqrt{3}</math> <span style="margin-left: 100px;"><math>(b = 7, c = -4)</math></span></p>	<p>B1 (1)</p> <p>M1</p> <p>A1, A1 (3)</p> <p style="text-align: right;"><b>4</b></p>
	<p>(a) <math>\pm 6\sqrt{3}</math> also scores B1.</p> <p>(b) M1: The 3 or 4 terms may be wrong.</p> <p style="margin-left: 20px;">1<sup>st</sup> A1 for 7, 2<sup>nd</sup> A1 for <math>-4\sqrt{3}</math>.</p> <p style="margin-left: 20px;">Correct answer <math>7 - 4\sqrt{3}</math> with no working scores all 3 marks.</p> <p style="margin-left: 20px;"><math>7 + 4\sqrt{3}</math> with or without working scores M1 A1 A0.</p> <p style="margin-left: 20px;">Other wrong answers with no working score no marks.</p>	

Question Number	Scheme	Marks
3.	<p>(a) </p> <p>Shape of <math>f(x)</math>            Moved up <math>\uparrow</math>            Asymptotes: <math>y = 3</math>  <math>x = 0</math> (Allow “y-axis”)            (<math>y \neq 3</math> is B0, <math>x \neq 0</math> is B0).</p> <p>(b) <math>\frac{1}{x} + 3 = 0</math> No variations accepted.  <math>x = -\frac{1}{3}</math> (or <math>-0.33 \dots</math>) Decimal answer requires at least 2 d.p.</p>	<p>B1            M1            B1            B1 (4)            M1            A1 (2)  <b>6</b></p>
	<p>(a) B1: Shape requires both branches and no obvious “overlap” with the asymptotes (see below), but otherwise this mark is awarded generously. The curve may, e.g., bend away from the asymptote a little at the end. Sufficient curve must be seen to suggest the asymptotic behaviour, both horizontal and vertical.            M1: Evidence of an upward translation parallel to the y-axis. The shape of the graph can be wrong, but the complete graph (both branches if they have 2 branches) must be translated upwards. This mark can be awarded generously by implication where the graph drawn is an upward translation of another standard curve (but <u>not</u> a straight line).            The B marks for asymptote equations are independent of the graph. Ignore extra asymptote equations, if seen.</p> <p>(b) Correct answer with no working scores both marks.            The answer may be seen on the sketch in part (a).            Ignore any attempts to find an intersection with the y-axis.</p> <p>e.g. </p> <p>(a) This scores B0 (clear overlap with horiz. asymp.)            M1 (Upward translation... bod that both branches have been translated).</p> <p>No marks unless the original curve is seen, to show upward translation.</p>	

Question Number	Scheme	Marks												
4.	$(x-2)^2 = x^2 - 4x + 4$ or $(y+2)^2 = y^2 + 4y + 4$ M: 3 or 4 terms $(x-2)^2 + x^2 = 10$ or $y^2 + (y+2)^2 = 10$ M: Substitute $2x^2 - 4x - 6 = 0$ or $2y^2 + 4y - 6 = 0$ Correct 3 terms $(x-3)(x+1) = 0, \quad x = \dots$ or $(y+3)(y-1) = 0, \quad y = \dots$ (The above factorisations may also appear as $(2x-6)(x+1)$ or equivalent). $x = 3 \quad x = -1$ or $y = -3 \quad y = 1$ $y = 1 \quad y = -3$ or $x = -1 \quad x = 3$ (Allow equivalent fractions such as: $x = \frac{6}{2}$ for $x = 3$ ).	M1 M1 A1 M1 A1 M1 A1      (7) 7												
	<p>1<sup>st</sup> M: ‘Squaring a bracket’, needs 3 or 4 terms, one of which must be an <math>x^2</math> or <math>y^2</math> term.</p> <p>2<sup>nd</sup> M: Substituting to get an equation in one variable (awarded generously).</p> <p>1<sup>st</sup> A: Accept equivalent forms, e.g. <math>2x^2 - 4x = 6</math>.</p> <p>3<sup>rd</sup> M: Attempting to solve a 3-term quadratic, to get 2 solutions.</p> <p>4<sup>th</sup> M: Attempting at least one <math>y</math> value (or <math>x</math> value).</p> <p>If <math>y</math> solutions are given as <math>x</math> values, or vice-versa, penalise at the end, so that it is possible to score M1 M1A1 M1 A1 M0 A0.</p> <p>Strict “pairing of values” at the end is <u>not</u> required.</p> <p>“Non-algebraic” solutions:</p> <p>No working, and only one correct solution pair found (e.g. <math>x = 3, y = 1</math>):                      M0 M0 A0 M0 A0 M1 A0</p> <p>No working, and both correct solution pairs found, but not demonstrated:                      M0 M0 A0 M1 A1 M1 A1</p> <p>Both correct solution pairs found, and demonstrated, perhaps in a table of values:                      Full marks</p> <p><u>Squaring individual terms:</u> e.g.</p> <table data-bbox="172 1630 1197 1816"> <tr> <td><math>y^2 = x^2 + 4</math></td> <td>M0</td> <td></td> </tr> <tr> <td><math>x^2 + 4 + x^2 = 10</math></td> <td>M1 A0</td> <td>(Eqn. in one variable)</td> </tr> <tr> <td><math>x = \sqrt{3}</math></td> <td>M0 A0</td> <td>(Not solving 3-term quad.)</td> </tr> <tr> <td><math>y^2 = x^2 + 4 = 7</math>      <math>y = \sqrt{7}</math></td> <td>M1 A0</td> <td>(Attempting one <math>y</math> value)</td> </tr> </table>	$y^2 = x^2 + 4$	M0		$x^2 + 4 + x^2 = 10$	M1 A0	(Eqn. in one variable)	$x = \sqrt{3}$	M0 A0	(Not solving 3-term quad.)	$y^2 = x^2 + 4 = 7$ $y = \sqrt{7}$	M1 A0	(Attempting one $y$ value)	
$y^2 = x^2 + 4$	M0													
$x^2 + 4 + x^2 = 10$	M1 A0	(Eqn. in one variable)												
$x = \sqrt{3}$	M0 A0	(Not solving 3-term quad.)												
$y^2 = x^2 + 4 = 7$ $y = \sqrt{7}$	M1 A0	(Attempting one $y$ value)												

Question Number	Scheme	Marks
5.	<p><u>Use</u> of <math>b^2 - 4ac</math>, perhaps implicit (e.g. in quadratic formula)</p> $(-3)^2 - 4 \times 2 \times -(k+1) < 0 \quad (9 + 8(k+1) < 0)$ $8k < -17 \quad (\text{Manipulate to get } pk < q, \text{ or } pk > q, \text{ or } pk = q)$ $k < -\frac{17}{8} \quad \left( \text{Or equiv : } k < -2\frac{1}{8} \text{ or } k < -2.125 \right)$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1cso (4)</p> <p style="text-align: right;"><b>4</b></p>
	<p>1<sup>st</sup> M: Could also be, for example, comparing or equating <math>b^2</math> and <math>4ac</math>. Must be considering the <u>given</u> quadratic equation. There must <u>not</u> be <math>x</math> terms in the expression, but there must be a <math>k</math> term.</p> <p>1<sup>st</sup> A: Correct expression (need not be simplified) and correct inequality sign. Allow also <math>-3^2 - 4 \times 2 \times -(k+1) &lt; 0</math>.</p> <p>2<sup>nd</sup> M: Condone sign or bracketing mistakes in manipulation. Not dependent on 1<sup>st</sup> M, but should not be given for irrelevant work. M0 M1 could be scored: e.g. where <math>b^2 + 4ac</math> is used instead of <math>b^2 - 4ac</math>.</p> <p><u>Special cases:</u></p> <ol style="list-style-type: none"> <li>Where there are <math>x</math> terms in the discriminant expression, but then division by <math>x^2</math> gives an inequality/equation in <math>k</math>. (This could score M0 A0 M1 A1).</li> <li>Use of <math>\leq</math> instead of <math>&lt;</math> loses one A mark only, at first occurrence, so an otherwise correct solution leading to <math>k \leq -\frac{17}{8}</math> scores M1 A0 M1 A1.</li> </ol> <p>N.B. Use of <math>b = 3</math> instead of <math>b = -3</math> implies no A marks.</p>	

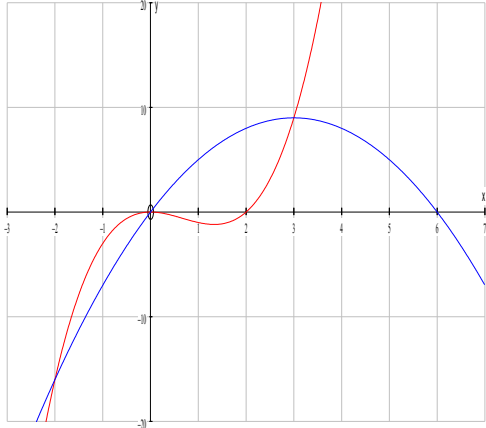




Question Number	Scheme	Marks
6.	<p>(a) <math>(4 + 3\sqrt{x})(4 + 3\sqrt{x})</math> seen, or a numerical value of <math>k</math> seen, (<math>k \neq 0</math>).            (The <math>k</math> value need not be explicitly stated... see below).  <math>16 + 24\sqrt{x} + 9x</math>, or <math>k = 24</math></p> <p>(b) <math>16 \rightarrow cx</math> or <math>kx^{1/2} \rightarrow cx^{3/2}</math> or <math>9x \rightarrow cx^2</math>  <math>\int(16 + 24\sqrt{x} + 9x)dx = 16x + \frac{9x^2}{2} + C, +16x^{3/2}</math></p>	<p>M1 A1cso (2)</p> <p>M1 A1, A1ft (3)</p> <p><b>5</b></p>
	<p>(a) e.g. <math>(4 + 3\sqrt{x})(4 + 3\sqrt{x})</math> alone scores M1 A0, (but <u>not</u> <math>(4 + 3\sqrt{x})^2</math> alone).            e.g <math>16 + 12\sqrt{x} + 9x</math> scores M1 A0.  <math>k = 24</math> or <math>16 + 24\sqrt{x} + 9x</math>, with no further evidence, scores full marks M1 A1.            Correct solution only (cso): any wrong working seen loses the A mark.</p> <p>(b) A1: <math>16x + \frac{9x^2}{2} + C</math>. Allow 4.5 or <math>4\frac{1}{2}</math> as equivalent to <math>\frac{9}{2}</math>.            A1ft: <math>\frac{2k}{3}x^{3/2}</math> (candidate's value of <math>k</math>, or general <math>k</math>).            For this final mark, allow for example <math>\frac{48}{3}</math> as equivalent to 16, but do <u>not</u> allow unsimplified "double fractions" such as <math>\frac{24}{(3/2)}</math>, and do <u>not</u> allow unsimplified "products" such as <math>\frac{2}{3} \times 24</math>.            A single term is required, e.g. <math>8x^{3/2} + 8x^{3/2}</math> is not enough.</p> <p>An otherwise correct solution with, say, <math>C</math> missing, followed by an incorrect solution including <math>+ C</math> can be awarded full marks (isw, but allowing the <math>C</math> to appear at any stage).</p>	

Question Number	Scheme	Marks
7.	<p>(a) <math>3x^2 \rightarrow cx^3</math> or <math>-6 \rightarrow cx</math> or <math>-8x^{-2} \rightarrow cx^{-1}</math></p> $f(x) = \frac{3x^3}{3} - 6x - \frac{8x^{-1}}{-1} \quad (+C) \quad \left( x^3 - 6x + \frac{8}{x} \right)$ <p>Substitute <math>x = 2</math> <u>and</u> <math>y = 1</math> into a 'changed function' to form an equation in <math>C</math>.</p> $1 = 8 - 12 + 4 + C \quad C = 1$ <p>(b) <math>3 \times 2^2 - 6 - \frac{8}{2^2}</math></p> $= 4$ <p>Eqn. of tangent: <math>y - 1 = 4(x - 2)</math></p> $y = 4x - 7 \quad (\text{Must be in this form})$	<p>M1</p> <p>A1 A1</p> <p>M1</p> <p>A1cso (5)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (4)</p> <p><b>9</b></p>
	<p>(a) First 2 A marks: <math>+C</math> is not required, and coefficients need <u>not</u> be simplified, but powers must be simplified.</p> <p>All 3 terms correct: A1 A1  Two terms correct: A1 A0  Only one term correct: A0 A0</p> <p>Allow the M1 A1 for finding <math>C</math> to be scored either in part (a) or in part (b).</p> <p>(b) 1<sup>st</sup> M: Substituting <math>x = 2</math> into <math>3x^2 - 6 - \frac{8}{x^2}</math> (must be this function).  2<sup>nd</sup> M: Awarded generously for attempting the equation of a straight line through (2, 1) or (1, 2) with any value of <math>m</math>, however found.  2<sup>nd</sup> M: Alternative is to use (2, 1) or (1, 2) in <math>y = mx + c</math> to <u>find a value</u> for <math>c</math>.</p> <p>If calculation for the gradient value is seen in part (a), it must be <u>used</u> in part (b) to score the first M1 A1 in (b).</p> <p><u>Using (1, 2) instead of (2, 1)</u>: Loses the 2<sup>nd</sup> method mark in (a).  Gains the 2<sup>nd</sup> method mark in (b).</p>	

Question Number	Scheme	Marks
8.	<p>(a) <math>4x \rightarrow k</math> or <math>3x^{3/2} \rightarrow kx^{1/2}</math> or <math>-2x^2 \rightarrow kx</math></p> $\frac{dy}{dx} = 4 + \frac{9}{2}x^{1/2} - 4x$ <p>(b) For <math>x = 4</math>, <math>y = (4 \times 4) + (3 \times 4\sqrt{4}) - (2 \times 16) = 16 + 24 - 32 = 8</math> (*)</p> <p>(c) <math>\frac{dy}{dx} = 4 + 9 - 16 = -3</math> M: Evaluate their <math>\frac{dy}{dx}</math> at <math>x = 4</math></p> <p>Gradient of normal = <math>\frac{1}{3}</math></p> <p>Equation of normal: <math>y - 8 = \frac{1}{3}(x - 4)</math>, <math>3y = x + 20</math> (*)</p> <p>(d) <math>y = 0</math>: <math>x = \dots (-20)</math> and use <math>(x_2 - x_1)^2 + (y_2 - y_1)^2</math></p> $PQ = \sqrt{24^2 + 8^2}$ or $PQ^2 = 24^2 + 8^2$ Follow through from $(k, 0)$ <p>May also be scored with <math>(-24)^2</math> and <math>(-8)^2</math>.</p> $= 8\sqrt{10}$	<p>M1</p> <p>A1 A1 (3)</p> <p>B1 (1)</p> <p>M1</p> <p>A1ft</p> <p>M1, A1 (4)</p> <p>M1</p> <p>A1ft</p> <p>A1 (3)</p> <p><b>11</b></p>
	<p>(a) For the 2 A marks coefficients need <u>not</u> be simplified, but powers must be simplified. For example, <math>\frac{3}{2} \times 3x^{1/2}</math> is acceptable.</p> <p>All 3 terms correct: A1 A1  Two terms correct: A1 A0  Only one term correct: A0 A0</p> <p>(b) There must be some evidence of the “24” value.</p> <p>(c) In this part, beware ‘working backwards’ from the given answer.</p> <p>A1ft: Follow through is just from the candidate’s <u>value</u> of <math>\frac{dy}{dx}</math>.</p> <p>2<sup>nd</sup> M: Is not given if an <math>m</math> value appears “from nowhere”.</p> <p>2<sup>nd</sup> M: Must be an attempt at a <u>normal</u> equation, not a tangent.</p> <p>2<sup>nd</sup> M: Alternative is to use <math>(4, 8)</math> in <math>y = mx + c</math> to <u>find a value</u> for <math>c</math>.</p> <p>(d) M: Using the normal equation to attempt coordinates of <math>Q</math>, (even if using <math>x = 0</math> instead of <math>y = 0</math>), <u>and</u> using Pythagoras to attempt <math>PQ</math> or <math>PQ^2</math>.</p> <p>Follow through from <math>(k, 0)</math>, but <u>not</u> from <math>(0, k)</math>...</p> <p>A common wrong answer is to use <math>x = 0</math> to give <math>\frac{20}{3}</math>. This scores M1 A0 A0.</p> <p>For final answer, accept other simplifications of <math>\sqrt{640}</math>, e.g. <math>2\sqrt{160}</math> or <math>4\sqrt{40}</math>.</p>	



Question Number	Scheme	Marks
9.	<p>(a) Recognising arithmetic series with first term 4 and common difference 3. (If not scored here, this mark may be given if seen elsewhere in the solution). <math>a + (n - 1)d = 4 + 3(n - 1) \quad (= 3n + 1)</math></p> <p>(b) <math>S_n = \frac{n}{2} \{2a + (n - 1)d\} = \frac{10}{2} \{8 + (10 - 1) \times 3\}, = 175,</math></p> <p>(c) <math>S_k &lt; 1750: \frac{k}{2} \{8 + 3(k - 1)\} &lt; 1750</math> (or <math>S_{k+1} &gt; 1750: \frac{k+1}{2} \{8 + 3k\} &gt; 1750</math>) <math>3k^2 + 5k - 3500 &lt; 0</math> (or <math>3k^2 + 11k - 3492 &gt; 0</math>) (Allow equivalent 3-term versions such as <math>3k^2 + 5k = 3500</math>). <math>(3k - 100)(k + 35) &lt; 0</math> Requires use of correct inequality throughout.(*)</p> <p>(d) <math>\frac{100}{3}</math> or equiv. <u>seen</u> (or <math>\frac{97}{3}</math>), <math>k = 33</math> (and no other values)</p>	<p>B1 M1 A1 (3)</p> <p>M1 A1, A1 (3)</p> <p>M1 M1 A1 A1cso (4) M1, A1 (2)</p> <p><b>12</b></p>
	<p>(a) B1: Usually identified by <math>a = 4</math> and <math>d = 3</math>. M1: Attempted use of term formula for arithmetic series, or... answer in the form <math>(3n + \text{constant})</math>, where the constant is a non-zero value. Answer for (a) does not require simplification, and a correct answer without working scores all 3 marks.</p> <p>(b) M1: Use of correct sum formula with <math>n = 9, 10</math> or <math>11</math>. A1: Correct, perhaps unsimplified, numerical version. A1: 175 <u>Alternative:</u> (Listing and summing terms). M1: Summing 9, 10 or 11 terms. (At least 1<sup>st</sup>, 2<sup>nd</sup> and last terms must be seen). A1: Correct terms (perhaps implied by last term 31). A1: 175 <u>Alternative:</u> (Listing all sums) M1: Listing 9, 10 or 11 sums. (At least 4, 7, ....., "last"). A1: Correct sums, correct finishing value 175. A1: 175 <u>Alternative:</u> (Using last term). M1: Using <math>S_n = \frac{n}{2}(a + l)</math> with <math>T_9, T_{10}</math> or <math>T_{11}</math> as the last term. A1: Correct numerical version <math>\frac{10}{2}(4 + 31)</math>. A1: 175 Correct answer with <u>no</u> working scores 1 mark: 1,0,0.</p> <p>(c) For the first 3 marks, allow <u>any inequality sign</u>, or <u>equals</u>. 1<sup>st</sup> M: Use of correct sum formula to form inequality or equation in <math>k</math>, with the 1750. 2<sup>nd</sup> M: (Dependent on 1<sup>st</sup> M). Form 3-term quadratic in <math>k</math>. 1<sup>st</sup> A: Correct 3 terms. Allow credit for part (c) if valid work is seen in part (d).</p> <p>(d) Allow both marks for <math>k = 33</math> seen without working. Working for part (d) must be seen in part (d), not part (c).</p>	

Question Number	Scheme	Marks
10.	<p>(a) </p> <p>(i) Shape  or  or   Max. at (0, 0).  (2, 0), (or 2 shown on x-axis).</p> <p>(ii) Shape   (It need not go below x-axis)  Through origin.  (6, 0), (or 6 shown on x-axis).</p> <p>(b) <math>x^2(x-2) = x(6-x)</math>  <math>x^3 - x^2 - 6x = 0</math> Expand to form 3-term cubic (or 3-term quadratic if divided by <math>x</math>), with all terms on one side. The “= 0” may be implied.  <math>x(x-3)(x+2) = 0</math> <math>x = \dots</math> Factor <math>x</math> (or divide by <math>x</math>), and solve quadratic.  <math>x = 3</math> and <math>x = -2</math>  <math>x = -2</math>: <math>y = -16</math> Attempt <math>y</math> value for a non-zero <math>x</math> value by substituting back into <math>x^2(x-2)</math> or <math>x(6-x)</math>.  <math>x = 3</math>: <math>y = 9</math> Both <math>y</math> values are needed for A1.  (-2, -16) and (3, 9)  (0, 0) This can just be written down. Ignore any ‘method’ shown. (But must be seen in part (b)).</p>	<p>B1  B1  B1 (3)  B1  B1  B1 (3)  M1  M1  M1  A1  M1  A1  B1 (7)  <b>13</b></p>
	<p>(a) (i) For the third ‘shape’ shown above, where a section of the graph coincides with the <math>x</math>-axis, the B1 for (2, 0) can still be awarded if the 2 is shown on the <math>x</math>-axis.  For the final B1 in (i), and similarly for (6, 0) in (ii):  There must be a sketch.  If, for example (2, 0) is written <u>separately</u> from the sketch, the sketch must not clearly contradict this.  If (0, 2) instead of (2, 0) is shown <u>on the sketch</u>, allow the mark.  Ignore extra intersections with the <math>x</math>-axis.  (ii) 2<sup>nd</sup> B is dependent on 1<sup>st</sup> B.  Separate sketches can score all marks.</p> <p>(b) Note the dependence of the first three M marks.  A common wrong solution is (-2, 0), (3, 0), (0, 0), which scores M0 A0 B1 as the last 3 marks.  A solution using <u>no</u> algebra (e.g. trial and error), can score up to 3 marks: M0 M0 M0 A0 M1 A1 B1. (The final A1 requires both <math>y</math> values).  Also, if the cubic is found but not solved algebraically, up to 5 marks: M1 M1 M0 A0 M1 A1 B1. (The final A1 requires both <math>y</math> values).</p>	